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Finding Impossible Differentials in ARX Ciphers under Weak Keys

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Background

Impossible differential (ID) attack is one of the most powerful cryptanalysis method in the field of symmetric ciphers. The methods to find IDs can be summarized in two phases:

- Phase 1: search IDs by treating the S-boxes as ideal ones, such as $\mathcal{U}$-method [KHL10], $\mathcal{UID}$-method [LLW14]
- Phase 2: search IDs by using DDT with automatic tools, such as based on MILP [ST17, CCJ+16], SAT/SMT [AK18, KLT15, MP13, RKJ+20] and CP [SGL+17]

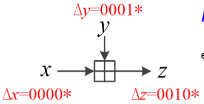
All methods above to find ID are based on two underlying assumptions:

- Markov cipher assumption
- key independence assumption

Motivation — Is Markov cipher assumption true?

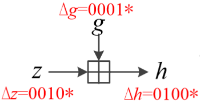
The trend to design ciphers towards lightweight: lighter round function and lighter key schedule. Take an example in ARX cipher as follows:

- Under Markov cipher assumption:



$Pr(\Delta x, \Delta y \rightarrow \Delta z) = p_1 > 0$

e.g. ($\Delta x = 00001$, $\Delta y = 00011$, $\Delta z = 00100$, $x = 00000$ and $y = 00000$)

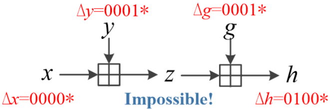


$Pr(\Delta z, \Delta e \rightarrow \Delta h) = p_2 > 0$

e.g. ($\Delta z = 00100$, $\Delta e = 00011$, $\Delta h = 01001$, $z = 00001$ and $e = 00000$)

$$Pr(\Delta x = 0000*, \Delta y = 0001*, \Delta g = 0001* \rightarrow \Delta h = 0100*) = p_1 p_2 > 0.$$

- Without Markov cipher assumption:



$$Pr(\Delta x = 0000*, \Delta y = 0001*, \Delta g = 0001* \rightarrow \Delta h = 0100*) = 0.$$

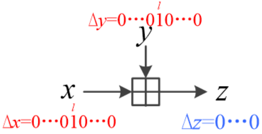
Properties on Single Addition Modulo 2^n

Property 1. [Li+19]

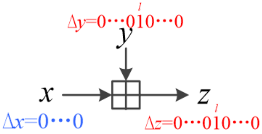
Let $x = z \boxplus y$ and $x' = z' \boxplus y'$, where $x, y, z, x', y', z' \in \mathbb{F}_2^n$. Suppose $\Delta x = x \oplus x'$, $\Delta y = y \oplus y'$ and $\Delta z = z \oplus z'$. If $\Delta x = \Delta y = 0 \cdots 0\overset{l}{1}0 \cdots 0$, then $\Delta z = 0 \cdots 0$ if and only if $x[l] \neq y[l]$ or $x'[l] \neq y'[l]$.

Property 2.

Let $x = z \boxminus y$ and $x' = z' \boxminus y'$, where $x, y, z, x', y', z' \in \mathbb{F}_2^n$. Suppose $\Delta x = x \oplus x'$, $\Delta y = y \oplus y'$ and $\Delta z = z \oplus z'$. If $\Delta z = \Delta y = \overset{n-1}{0} \cdots 0\overset{l}{1}0 \cdots \overset{0}{0}$, $0 \leq l < n-1$, then $\Delta x = 0 \cdots 0$ if and only if $z[l] = y[l]$ or $z'[l] = y'[l]$.



iff. $x[l] \neq y[l]$ or $x'[l] \neq y'[l]$



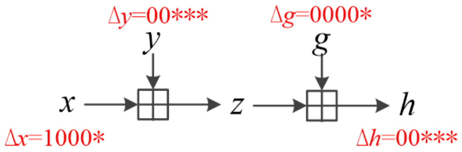
iff. $z[l] = y[l]$ or $z'[l] = y'[l]$

Properties on Two Consecutive Modular Additions

Property 3.

Let $z = x \boxplus y$, $z' = x' \boxplus y'$, $h = z \boxplus g$ and $h' = z' \boxplus g'$, where $x, y, z, g, h, x', y', z', g', h' \in \mathbb{F}_2^5$. Suppose $\Delta x = x \oplus x'$, $\Delta y = y \oplus y'$, $\Delta z = z \oplus z'$, $\Delta g = g \oplus g'$ and $\Delta h = h \oplus h'$. If $\Delta z[2:1] \neq 00$, then we have

$$(\Delta x = 1000*, \Delta y = 00***, \Delta g = 0000* \nrightarrow \Delta h = 00***).$$



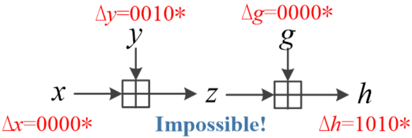
- When $\Delta z[2:1] \neq 00$, the differential will be impossible.
- In practical ciphers, $\Delta z[2:1] \neq 00$ is possible to happen.

Properties on Two Consecutive Modular Additions

Property 4.

Let $z = x \boxplus y$, $z' = x' \boxplus y'$, $h = z \boxplus g$ and $h' = z' \boxplus g'$, where $x, y, z, g, h, x', y', z', g', h' \in \mathbb{F}_2^5$. Suppose that $\Delta x = x \oplus x'$, $\Delta y = y \oplus y'$, $\Delta z = z \oplus z'$, $\Delta g = g \oplus g'$ and $\Delta h = h \oplus h'$. Then

$$(\Delta x = 0000*, \Delta y = 0010*, \Delta g = 0000* \nrightarrow \Delta h = 1010*).$$



- The carries brought by lower bits do not make the ID transitions viable.
- The ID can be extended to

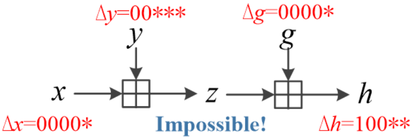
$$(\Delta x = * \cdots * \underbrace{\boxed{0000*}}^{i+4,\cdots,i} * \cdots *, \Delta y = * \cdots * \underbrace{\boxed{0010*}}^{i+4,\cdots,i} * \cdots *, \Delta g = * \cdots * \underbrace{\boxed{0000*}}^{i+4,\cdots,i} \nrightarrow \Delta h = * \cdots * \underbrace{\boxed{1010*}}^{i+4,\cdots,i} * \cdots *).$$

Properties on Two Consecutive Modular Additions

Property 5.

Let $z = x \boxplus y$, $z' = x' \boxplus y'$, $h = z \boxplus g$ and $h' = z' \boxplus g'$, where $x, y, z, g, h, x', y', z', g', h' \in \mathbb{F}_2^5$. Suppose that $\Delta x = x \oplus x'$, $\Delta y = y \oplus y'$, $\Delta z = z \oplus z'$, $\Delta g = g \oplus g'$ and $\Delta h = h \oplus h'$. Then

$$(\Delta x = 0000*,\ \Delta y = 00***, \Delta g = 0000* \nrightarrow \Delta h = 100**)$$

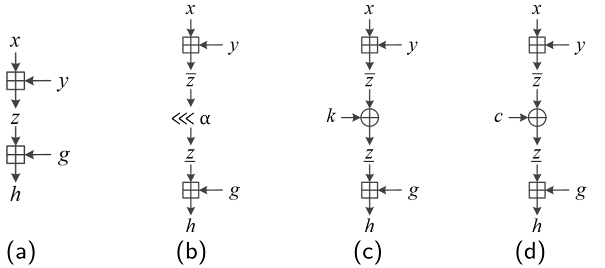


- The carries brought by lower bits do not make the ID transitions viable.
- The ID can be extended to

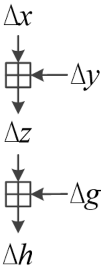
$$(\Delta x = *\cdots*\underbrace{0000*}^{i+4,\cdots,i}*\cdots*,\ \Delta y = *\cdots*\underbrace{00***}^{i+4,\cdots,i}*\cdots*,\ \Delta g = *\cdots*\underbrace{0000*}^{i+4,\cdots,i} \nrightarrow \Delta h = *\cdots*\underbrace{100**}^{i+4,\cdots,i}*\cdots*).$$

Summary on Properties 3 ∼ 5

- The ID patterns in Properties 3∼5 can be extended by adding uncertain bits on higher and lower bit positions.
- Properties 3∼5 represent just a thin selection of thousand ID patterns found experimentally.
- These Properties can be used to find IDs on four local constructions extracted from ARX ciphers.



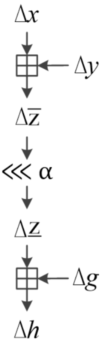
IDs on Local Construction (a)



(a)

Constraints	$\Delta z[i+2:i+1] \neq 00$		
Differentials	$\Delta x = (\ast\cdots\ast\ \overset{i+4,\cdots,i}{\boxed{1000\ast}}\ \ast\cdots\ast)$ $\Delta y = (\ast\cdots\ast\ \overset{i+4,\cdots,i}{\boxed{00\ast\ast\ast}}\ \ast\cdots\ast)$ $\Delta z = (\ast\cdots\ast\ \overset{i+4,\cdots,i}{\boxed{\ast\ast\ast\ast\ast}}\ \ast\cdots\ast)$ $\Delta g = (\ast\cdots\ast\ \overset{i+4,\cdots,i}{\boxed{0000\ast}}\ \ast\cdots\ast)$ $\Delta h = (\ast\cdots\ast\ \overset{i+4,\cdots,i}{\boxed{00\ast\ast\ast}}\ \ast\cdots\ast)$	$\Delta x = (\ast\cdots\ast\ \overset{i+4,\cdots,i}{\boxed{0000\ast}}\ \ast\cdots\ast)$ $\Delta y = (\ast\cdots\ast\ \overset{i+4,\cdots,i}{\boxed{0010\ast}}\ \ast\cdots\ast)$ $\Delta z = (\ast\cdots\ast\ \overset{i+4,\cdots,i}{\boxed{\ast\ast\ast\ast\ast}}\ \ast\cdots\ast)$ $\Delta g = (\ast\cdots\ast\ \overset{i+4,\cdots,i}{\boxed{0000\ast}}\ \ast\cdots\ast)$ $\Delta h = (\ast\cdots\ast\ \overset{i+4,\cdots,i}{\boxed{1010\ast}}\ \ast\cdots\ast)$	$\Delta x = (\ast\cdots\ast\ \overset{i+3,\cdots,i}{\boxed{0000}}\ \ast\cdots\ast)$ $\Delta y = (\ast\cdots\ast\ \overset{i+3,\cdots,i}{\boxed{00\ast\ast}}\ \ast\cdots\ast)$ $\Delta z = (\ast\cdots\ast\ \overset{i+3,\cdots,i}{\boxed{\ast\ast\ast\ast}}\ \ast\cdots\ast)$ $\Delta g = (\ast\cdots\ast\ \overset{i+3,\cdots,i}{\boxed{0000}}\ \ast\cdots\ast)$ $\Delta h = (\ast\cdots\ast\ \overset{i+3,\cdots,i}{\boxed{100\ast}}\ \ast\cdots\ast)$
Result	$(\Delta x,\ \Delta y,\ \Delta g \nrightarrow \Delta h)$ according to Property 3	$(\Delta x,\ \Delta y,\ \Delta g \nrightarrow \Delta h)$ according to Property 4	$(\Delta x,\ \Delta y,\ \Delta g \nrightarrow \Delta h)$ according to Property 5

IDs on Local Construction (b)

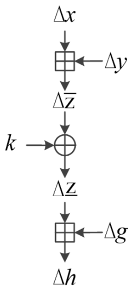


(b)

Constraints	$\Delta\bar{z}[i+2:i+1] \neq 00$		
Differentials	$\Delta x = (*\cdots* \overset{i+4,\cdots,i}{\boxed{1000*}} *\cdots*)$	$\Delta x = (*\cdots* \overset{i+4,\cdots,i}{\boxed{0000*}} *\cdots*)$	$\Delta x = (*\cdots* \overset{i+3,\cdots,i}{\boxed{0000}} *\cdots*)$
	$\Delta y = (*\cdots* \overset{i+4,\cdots,i}{\boxed{00***}} *\cdots*)$	$\Delta y = (*\cdots* \overset{i+4,\cdots,i}{\boxed{0010*}} *\cdots*)$	$\Delta y = (*\cdots* \overset{i+3,\cdots,i}{\boxed{00**}} *\cdots*)$
	$\Delta\bar{z} = (*\cdots* \overset{i+4,\cdots,i}{\boxed{*****}} *\cdots*)$	$\Delta\bar{z} = (*\cdots* \overset{i+4,\cdots,i}{\boxed{*****}} *\cdots*)$	$\Delta\bar{z} = (*\cdots* \overset{i+3,\cdots,i}{\boxed{****}} *\cdots*)$
	$\Delta\underline{z} = (*\cdots* \overset{j+4,\cdots,j}{\boxed{*****}} *\cdots*)$	$\Delta\underline{z} = (*\cdots* \overset{j+4,\cdots,j}{\boxed{*****}} *\cdots*)$	$\Delta\underline{z} = (*\cdots* \overset{j+3,\cdots,j}{\boxed{****}} *\cdots*)$
	$\Delta g = (*\cdots* \overset{j+4,\cdots,j}{\boxed{0000*}} *\cdots*)$	$\Delta g = (*\cdots* \overset{j+4,\cdots,j}{\boxed{0000*}} *\cdots*)$	$\Delta g = (*\cdots* \overset{j+3,\cdots,j}{\boxed{0000}} *\cdots*)$
	$\Delta h = (*\cdots* \overset{j+4,\cdots,j}{\boxed{00***}} *\cdots*)$	$\Delta h = (*\cdots* \overset{j+4,\cdots,j}{\boxed{1010*}} *\cdots*)$	$\Delta h = (*\cdots* \overset{j+3,\cdots,j}{\boxed{100*}} *\cdots*)$
Result	$(\Delta x, \Delta y, \Delta g \nrightarrow \Delta h)$ according to Property 3	$(\Delta x, \Delta y, \Delta g \nrightarrow \Delta h)$ according to Property 4	$(\Delta x, \Delta y, \Delta g \nrightarrow \Delta h)$ according to Property 5

- The i-th bit of $\bar{z}$ is cyclically shifted to the j-th bit of $\underline{z}$.

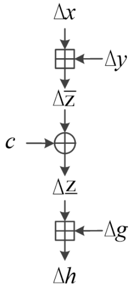
IDs on Local Construction (c)



(c)

Constraints	$k[i+3:i+1] = 000$ or 111; $\Delta\bar{z}[i+2:i+1] \neq 00$	$k[i+3:i+2] = 00$ or 11	$k[i+2:i+1] = 00$ or 11
Differentials	$\Delta x = (*\cdots*\ \overset{i+4,\cdots,i}{\boxed{1000*}}\ *\cdots*)$	$\Delta x = (*\cdots*\ \overset{i+4,\cdots,i}{\boxed{0000*}}\ *\cdots*)$	$\Delta x = (*\cdots*\ \overset{i+3,\cdots,i}{\boxed{0000}}\ *\cdots*)$
	$\Delta y = (*\cdots*\ \overset{i+4,\cdots,i}{\boxed{00***}}\ *\cdots*)$	$\Delta y = (*\cdots*\ \overset{i+4,\cdots,i}{\boxed{0010*}}\ *\cdots*)$	$\Delta y = (*\cdots*\ \overset{i+3,\cdots,i}{\boxed{00**}}\ *\cdots*)$
	$\Delta z = (*\cdots*\ \overset{i+4,\cdots,i}{\boxed{*****}}\ *\cdots*)$	$\Delta z = (*\cdots*\ \overset{i+4,\cdots,i}{\boxed{*****}}\ *\cdots*)$	$\Delta z = (*\cdots*\ \overset{i+3,\cdots,i}{\boxed{****}}\ *\cdots*)$
	$\Delta g = (*\cdots*\ \overset{i+4,\cdots,i}{\boxed{0000*}}\ *\cdots*)$	$\Delta g = (*\cdots*\ \overset{i+4,\cdots,i}{\boxed{0000*}}\ *\cdots*)$	$\Delta g = (*\cdots*\ \overset{i+3,\cdots,i}{\boxed{0000}}\ *\cdots*)$
	$\Delta h = (*\cdots*\ \overset{i+4,\cdots,i}{\boxed{00***}}\ *\cdots*)$	$\Delta h = (*\cdots*\ \overset{i+4,\cdots,i}{\boxed{1010*}}\ *\cdots*)$	$\Delta h = (*\cdots*\ \overset{i+3,\cdots,i}{\boxed{100*}}\ *\cdots*)$
Result	$(\Delta x, \Delta y, \Delta g \nrightarrow \Delta h)$ according to Property 3	$(\Delta x, \Delta y, \Delta g \nrightarrow \Delta h)$ according to Property 4	$(\Delta x, \Delta y, \Delta g \nrightarrow \Delta h)$ according to Property 5

IDs on Local Construction (d)



(d)

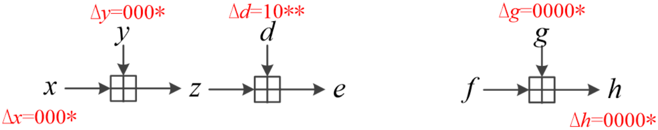
Constraints	$c[i+3:i+1] = 000$ or 111 $\Delta\overline{z}[i+2:i+1] \neq 00$	$c[i+3:i+2] = 00$ or 11	$c[i+2:i+1] = 00$ or 11
Differentials	$\Delta x = (*\cdots* \overset{i+4,\cdots,i}{\boxed{1000*}} *\cdots*)$	$\Delta x = (*\cdots* \overset{i+4,\cdots,i}{\boxed{0000*}} *\cdots*)$	$\Delta x = (*\cdots* \overset{i+3,\cdots,i}{\boxed{0000}} *\cdots*)$
	$\Delta y = (*\cdots* \overset{i+4,\cdots,i}{\boxed{00***}} *\cdots*)$	$\Delta y = (*\cdots* \overset{i+4,\cdots,i}{\boxed{0010*}} *\cdots*)$	$\Delta y = (*\cdots* \overset{i+3,\cdots,i}{\boxed{00**}} *\cdots*)$
	$\Delta z = (*\cdots* \overset{i+4,\cdots,i}{\boxed{*****}} *\cdots*)$	$\Delta z = (*\cdots* \overset{i+4,\cdots,i}{\boxed{*****}} *\cdots*)$	$\Delta z = (*\cdots* \overset{i+3,\cdots,i}{\boxed{****}} *\cdots*)$
	$\Delta g = (*\cdots* \overset{i+4,\cdots,i}{\boxed{0000*}} *\cdots*)$	$\Delta g = (*\cdots* \overset{i+4,\cdots,i}{\boxed{0000*}} *\cdots*)$	$\Delta g = (*\cdots* \overset{i+3,\cdots,i}{\boxed{0000}} *\cdots*)$
	$\Delta h = (*\cdots* \overset{i+4,\cdots,i}{\boxed{00***}} *\cdots*)$	$\Delta h = (*\cdots* \overset{i+4,\cdots,i}{\boxed{1010*}} *\cdots*)$	$\Delta h = (*\cdots* \overset{i+3,\cdots,i}{\boxed{100*}} *\cdots*)$
Result	$(\Delta x, \Delta y, \Delta g \nrightarrow \Delta h)$ according to Property 3	$(\Delta x, \Delta y, \Delta g \nrightarrow \Delta h)$ according to Property 4	$(\Delta x, \Delta y, \Delta g \nrightarrow \Delta h)$ according to Property 5

Properties on Three Consecutive Modular Additions

Property 6.

$x \boxplus y = z \pmod{2^4}$, $x' \boxplus y' = z' \pmod{2^4}$, $z \boxplus d = e \pmod{2^4}$, $z' \boxplus d' = e' \pmod{2^4}$, $f \boxplus g = h \pmod{2^5}$ and $f' \boxplus g' = h' \pmod{2^5}$. Suppose that $\Delta x = x \oplus x'$, $\Delta y = y \oplus y'$, $\Delta z = z \oplus z'$, $\Delta d = d \oplus d'$, $\Delta e = e \oplus e'$, $\Delta f = f \oplus f'$, $\Delta g = g \oplus g'$ and $\Delta h = h \oplus h'$. If $f[4:1] = e$, then

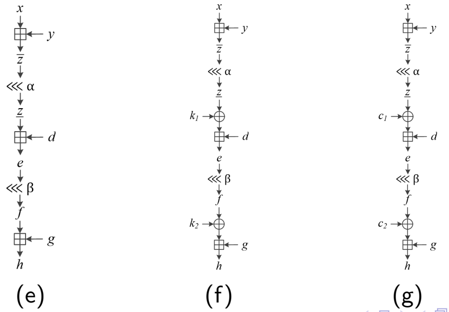
$$(\Delta x = 000*, \Delta y = 000*, \Delta d = 10**, \Delta g = 0000* \nrightarrow \Delta h = 0000*)$$



- When $f = e \| *$, the differential will be impossible.
- In practical ciphers, $f = e \| *$ is possible to happen.

Local Constructions of ARX ciphers

- The ID patterns in Property 6 can be extended by adding uncertain bits on higher and lower bit positions.
- The structures of consecutive three modular additions and its variants are extracted from ARX ciphers.
- These Property 6 can be used to find IDs on these local constructions below. Please refer to the table on the next page.



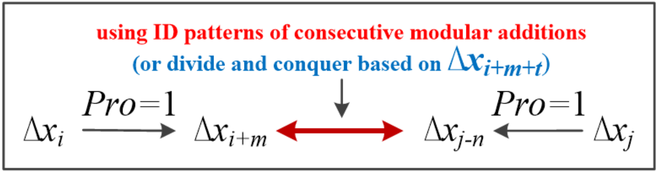
IDs on Local Constructions (e)∼ (g)

Constraints		$k_1[2:1] = 00$ or 11 $k_2[j+3:j+1] = 000$ or 111	$c_1[2:1] = 00$ or 11 $c_2[j+3:j+1] = 000$ or 111
Differentials	$\Delta x = (*\cdots* \overset{i+3,\cdots,i}{\boxed{000*}} *\cdots*)$	$\Delta x = (*\cdots* \overset{i+3,\cdots,i}{\boxed{000*}} *\cdots*)$	$\Delta x = (*\cdots* \overset{i+3,\cdots,i}{\boxed{000*}} *\cdots*)$
	$\Delta y = (*\cdots* \overset{i+3,\cdots,i}{\boxed{000*}} *\cdots*)$	$\Delta y = (*\cdots* \overset{i+3,\cdots,i}{\boxed{000*}} *\cdots*)$	$\Delta y = (*\cdots* \overset{i+3,\cdots,i}{\boxed{000*}} *\cdots*)$
	$\Delta \bar{z} = (*\cdots* \overset{i+3,\ \cdots,\ i}{\boxed{*\ ***}} *\cdots*)$	$\Delta \bar{z} = (*\cdots* \overset{i+3,\ \cdots,\ i}{\boxed{*\ ***}} *\cdots*)$	$\Delta \bar{z} = (*\cdots* \overset{i+3,\ \cdots,\ i}{\boxed{*\ ***}} *\cdots*)$
	$\Delta \underline{z} = (*\cdots* \overset{3,\ \cdots,\ 0}{\boxed{****}})$	$\Delta \underline{z} = (*\cdots* \overset{3,\ \cdots,\ 0}{\boxed{****}})$	$\Delta \underline{z} = (*\cdots* \overset{3,\ \cdots,\ 0}{\boxed{****}})$
	$\Delta d = (*\cdots* \overset{3,\ \cdots,\ 0}{1\boxed{0**}})$	$\Delta d = (*\cdots* \overset{3,\ \cdots,\ 0}{1\boxed{0**}})$	$\Delta d = (*\cdots* \overset{3,\ \cdots,\ 0}{1\boxed{0**}})$
	$\Delta e = (*\cdots* \overset{3,\ \cdots,\ 0}{\boxed{****}})$	$\Delta e = (*\cdots* \overset{3,\ \cdots,\ 0}{\boxed{****}})$	$\Delta e = (*\cdots* \overset{3,\ \cdots,\ 0}{\boxed{****}})$
	$\Delta f = (*\cdots* \overset{j+4,\ \cdots,\ j}{\boxed{*****}} *\cdots*)$	$\Delta f = (*\cdots* \overset{j+4,\ \cdots,\ j}{\boxed{*****}} *\cdots*)$	$\Delta f = (*\cdots* \overset{j+4,\ \cdots,\ j}{\boxed{*****}} *\cdots*)$
	$\Delta g = (*\cdots* \overset{j+4,\cdots,j}{\boxed{0000*}} *\cdots*)$	$\Delta g = (*\cdots* \overset{j+4,\cdots,j}{\boxed{0000*}} *\cdots*)$	$\Delta g = (*\cdots* \overset{j+4,\cdots,j}{\boxed{0000*}} *\cdots*)$
	$\Delta h = (*\cdots* \overset{j+4,\cdots,j}{\boxed{0000*}} *\cdots*)$	$\Delta h = (*\cdots* \overset{j+4,\cdots,j}{\boxed{0000*}} *\cdots*)$	$\Delta h = (*\cdots* \overset{j+4,\cdots,j}{\boxed{0000*}} *\cdots*)$
Result	(Δx, Δy, Δd, $\Delta g \nrightarrow \Delta h$) according to Property 6		

[1] The i-th bit of $\bar{z}$ is cyclically shifted to LSB of $\underline{z}$.

[2] The LSB of e is cyclically shifted to the j-th bit of f.

Framework for Finding IDs in ARXs under weak keys



Step 1. Obtain the differentials $\Delta x_i \rightarrow \Delta x_{i+m}$ and $\Delta x_{j-n} \leftarrow \Delta x_j$ by some tool according to properties of addition modulo 2^n.

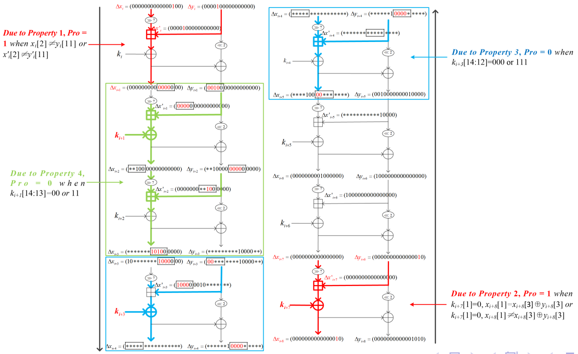
Step 2. Check the possibility of the differential $\Delta x_{i+m} \rightarrow \Delta x_{j-n}$ by using ID patterns of consecutive modular additions.
If $\Delta x_{i+m} \nrightarrow \Delta x_{j-n}$, return $\Delta x_i \nrightarrow \Delta x_j$.

Step 3. Use some tool to obtain possible forms of intermediate difference Δx_{i+m+t} and **divide and conquer** with them. Specially, return to Step 2 to check $\Delta x_{i+m} \rightarrow \Delta x_{i+m+t}$ and $\Delta x_{i+m+t} \rightarrow \Delta x_{j-n}$.
$(i+m < i+m+t < j-n)$

Apply to SPECK32/64

When $k_{i+1}[14:13] = 00$ (or 11), $k_{i+3}[14:12] = 000$ (or 111), $x_i[2] \neq y_i[11]$ or $(x'_i[2] \neq y'_i[11]$, there are two 8-round IDs for SPECK32/64 under 2^{60} weak keys:

- $(\Delta x_i = 0\cdots0100, \Delta y_i = 000010\cdots0) \nrightarrow (\Delta x_{i+8} = 0\cdots010, \Delta y_{i+8} = 0\cdots01010)$ under $k_{i+7}[1] = 0$ if $x_{i+8}[2] = x_{i+8}[4] \oplus y_{i+8}[4]$.
- $(\Delta x_i = 0\cdots0100, \Delta y_i = 000010\cdots0) \nrightarrow (\Delta x_{i+8} = 0\cdots010, \Delta y_{i+8} = 0\cdots01010)$ under $k_{i+7}[1] = 1$ if $x_{i+8}[2] \neq x_{i+8}[4] \oplus y_{i+8}[4]$.



Apply to LEA-k ($k = 128, 192, 256$)

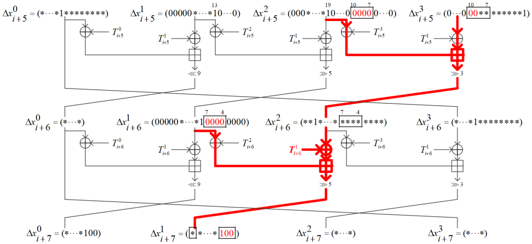
11-round ID for LEA-k under 2^{k-1} weak keys:

- Rounds $i \sim i+5$:

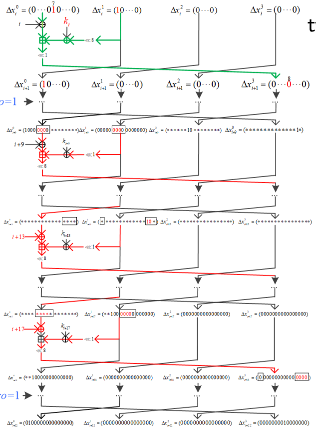
  $(10\cdots0, 10\cdots0, 10\cdots0, 10\cdots0) \rightarrow (*\cdots*\overset{9}{1}*\cdots*, 0\cdots\overset{27}{0}*\cdots*\overset{13}{1}0\cdots0, 000*\cdots*\overset{19}{1}0\cdots0, 0\cdots\overset{9}{0}*\cdots*)$

  with prob. 1
- Rounds $i+7 \sim i+11$:

  $(*\cdots*100, **\cdots*100, *\cdots*, *\cdots*) \rightarrow (0\cdots0, 0\cdots0, 00010\cdots0, 0\cdots0)$ with prob. 1
- Rounds $i+5 \sim i+6$: When $T^1_{i+6}[6:5] = 00$ or 11, the differential of **the red part is impossible according to the Property 5**.



Apply to CHAM64/128



two 22-round IDs for CHAM-64/128 under 2^{127} weak keys:

- $(\Delta x_i^0 = 0\cdots0\overset{7}{1}0\cdots0, \Delta x_i^1 = 10\cdots0, \Delta x_i^2 = 0\cdots0, \Delta x_i^3 = 0\cdots0)$
  $\nrightarrow (\Delta x_{i+22}^0 = 01\cdots0, \Delta x_{i+22}^1 = 0\cdots0, \Delta x_{i+22}^2 = 0\cdots0, \Delta x_{i+22}^3 = 0\cdots0\overset{7}{1}0)$
  under $k_i[7] = 0$ if $x_i^0[7] \neq x_i^1[15]$.
- $(\Delta x_i^0 = 0\cdots0\overset{7}{1}0\cdots0, \Delta x_i^1 = 10\cdots0, \Delta x_i^2 = 0\cdots0, \Delta x_i^3 = 0\cdots0)$
  $\nrightarrow (\Delta x_{i+22}^0 = 01\cdots0, \Delta x_{i+22}^1 = 0\cdots0, \Delta x_{i+22}^2 = 0\cdots0, \Delta x_{i+22}^3 = 0\cdots0\overset{7}{1}0)$
  under $k_i[7] = 1$ if $x_i^0[7] = x_i^1[15]$.
- **According to Property 1**, if $k_i[7] = 0$, $x_i^0[7] \neq x_i^1[15]$ or $k_i[7] = 1$, $x_i^0[7] = x_i^1[15]$, there is the differential $(\Delta x_i \rightarrow \Delta x_{i+1})$**with Probability 1**, refer to the green part.
- When $(i+13)[2:1] = 00$ or 11 and $(i+17)[10:8] = 000$ or 111, the differential $(\Delta x_{i+9} \rightarrow \Delta x_{i+18})$of **the red part is impossible according to the property 6**.

Compare with Previous Results

Cipher	Round	Weak key space	Starting round	Reference
SPECK-32/64	6	2^{64}	any	[Li+18]
	6	2^{64}	any	[XSQ17]
	7	2^{64}	any	[Li+19]
	8	$\mathbf{2^{60}}$	**any**	This work
LEA-k	10	2^{k}	any	[Hon+14]
	10	2^{k}	any	[Cui+16]
	11	$\mathbf{2^{k-1}}$	**any**	This work
CHAM-64/128	18	2^{128}	any	[Koo+17]
	20	2^{128}	$i, i \in A$	[Xu+22]
	22	$\mathbf{2^{127}}$	$\mathbf{i}, \mathbf{i} \in \mathbf{B}$	This work

1 $A = \{3, 5, 11, 13, 19, 21, 27, 29, 35, 37, 43, 45, 51, 53, 59\}$.

2 $B = \{2, 4, 10, 12, 18, 20, 26, 28, 34, 36, 42, 44, 50, 52, 58\}$.

Conclusion

This work

- Some more accurate differentials properties on consecutive addition modulo 2^n.
- A framework to find IDs of ARX ciphers under weak key.
- Apply to SPECK, LEA and CHAM to find longer IDs under weak key.

Future work

- As properties $3 \sim 6$ represent just a thin selection of the ID patterns found experimentally, it is valuable to continue analyzing these ID patterns.
- It is also a meaningful work to try to build an automated search model to find more impossible differentials.
- It is worthwhile to dig deeper for more impossible differentials to get better key recovery attacks for ARX ciphers.

Thanks for your attention!

Q & A