Analysis of AES, SKINNY, and Others with Constraint Programming

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Outline

- Constraint programming (CP)
- Automatic cryptanalysis with CP
- Comparing solvers
- Conclusion and Discussion

Constraint Programming

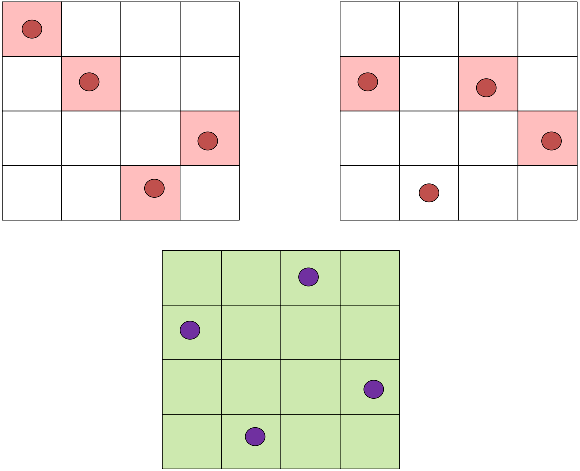
Definition : CP and CSP

CP is used to solve Constraint Satisfaction Problems (CSPs). A CSP is defined by a triple (X, D, C) such that

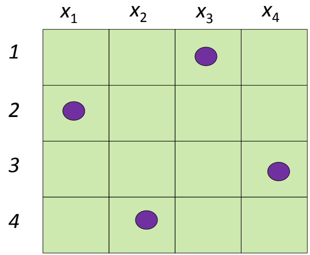
- $X = \{x_1, \cdots, x_n\}$ is a finite set of variables
- $D = \{D_1, \cdots, D_n\}$, where D_i is the domain of x_i, that is, the finite set of values that may be assigned to x_i. Hence $x_i \in D_i$.
- $C = \{C_1, \cdots, C_m\}$ is a set of constraints, where C_i defines a relation over $scope(C_i) \subseteq X$ which restrict the set of values that may be assigned simultaneously to these variables.

Constraint Programming – The *n* Queens Problem

Place n queens on an chessboard such that no queen can attack any other.



Formulating the n-Queens Problem



- Variables : $X = \{x_1, x_2, x_3, x_4\}$, x_i represents the row number of the queen at ith col
- Domains : $D = \{D_1, D_2, D_3, D_4\}$ where $D_i = \{1, 2, 3, 4\}$
- Constraints : $x_i \neq x_j$, $|x_i - x_{i+j}| \neq j$

Declare the constraints in extension

$(x_1, x_2) \in \{(1,3), (1,4), (2,4)(3,1), (4,1), (4,2)\}$
$(x_1, x_3) \in \{\cdots\}$

Constraint Programming : how to solve ?

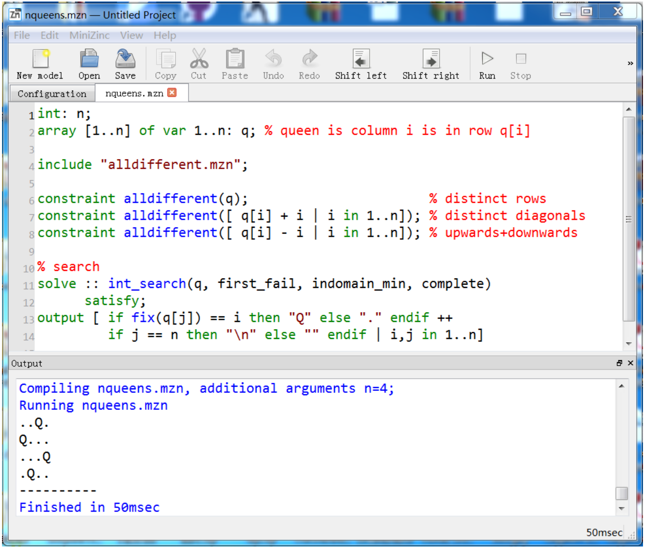
- Step 1. input the variables, domains, and constraints into a CP solver (Declare the problem)
- Step 2 : Wait for the solution

CP Solvers

- The CP solvers implement sophisticated backtracking and inference (constraint propagation) algorithms to find a solution.
- Solvers
  - Dedicated CP solvers : Choco, Chuffed, Gecode ...
  - SAT, MILP or hybrid solvers
  - Standard modelling language : Minizinc.

Eugene C. Freuder, April 1997

Constraint programming represents one of the closest approaches computer science has yet made to the Holy Grail of programming : the user states the problem, the computer solves it.


nqueens.mzn — Untitled Project
File Edit MiniZinc View Help
New model Open Save Copy Cut Paste Undo Redo Shift left Shift right Run Stop
Configuration nqueens.mzn
int: n;
array [1..n] of var 1..n: q; % queen is column i is in row q[i]
include "alldifferent.mzn";
constraint alldifferent(q); % distinct rows
constraint alldifferent([q[i] + i | i in 1..n]); % distinct diagonals
constraint alldifferent([q[i] - i | i in 1..n]); % upwards+downwards
% search
solve :: int_search(q, first_fail, indomain_min, complete)
satisfy;
output [if fix(q[j]) == i then "Q" else "." endif ++
if j == n then "\n" else "" endif | i,j in 1..n]
Output
Compiling nqueens.mzn, additional arguments n=4;
Running nqueens.mzn
..Q.
Q...
...Q
.Q..

Finished in 50msec
50msec

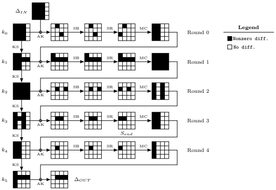
Automatic Cryptanalysis of Symmetric-key Algorithms

- Search algorithms implemented from scratch in general-purpose programming languages
- SAT/SMT based methods
- Mixed-integer programming (MILP) based methods
- Constraint programming (CP) based methods

Advantages of the CP approach

- Easy to implement
- Modelling process of CP is much more straightforward : input allowed tuples directly
- directly benefit from the advances in the resolution technique

Search for related-key differential characteristics of AES-128

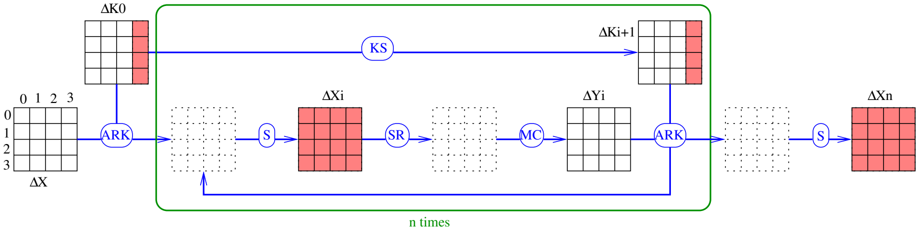


Related work

- [Alex Biryukov and Ivica Nikolić, EUROCRYPT 2010]
- [Pierre-Alain Fouque, Jérémy Jean and Thomas Peyrin, CRYPTO 2013]
- [David Gerault, Marine Minier and Christine Solnon, CP 2016]

- Step 1 : Find truncated differential characteristics with the minimum number of active S-boxes
- Step 2 : Instantiate the truncated differential characteristics with actual differences

CP Model for Step 1 : Variables and Constraints



- 0-1 variables
  - $\Delta X[j][k]$
  - $\Delta X_i[j][k]$
  - $\Delta Y_i[j][k]$
  - $\Delta K_i[j][k]$
- Constraints
  - ARK
  - SR-MC
  - KS
  - XOR

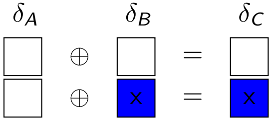
Semantics of the variables

These variables are used to trace the propagation of the truncated differences.

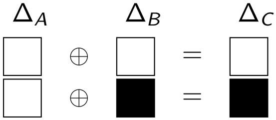
XOR Constraint

(white $= 0$, colored $\neq 0$)

Byte values



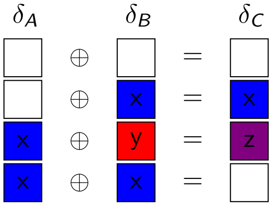
Boolean abstraction



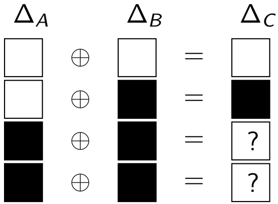
XOR Constraint

(white $=0$, colored $\neq 0$)

Byte values



Boolean abstraction

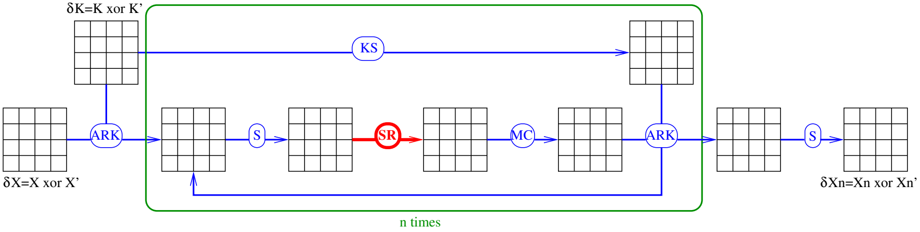


Δ_A	Δ_B	Δ_C
0	0	0
0	1	1
1	0	1
1	1	?

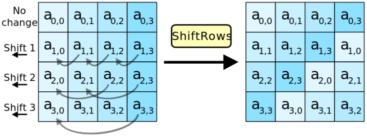
Definition of the XOR constraint

$\Delta_A + \Delta_B + \Delta_C \neq 1$

SR-MC Constraint



At byte level

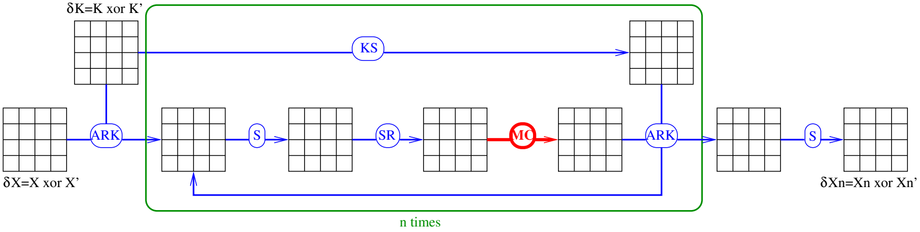


Definition of the SR-MC constraint

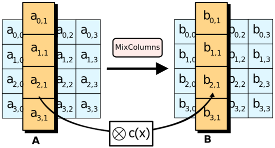
$\forall j \in [0;3]$:

$$\sum_{k=0}^{3} \Delta X_i[(k+j)\%4][k] + \Delta Y_i[j][k] \in \{0,5,6,7,8\}$$

SR-MC Constraint



At byte level



MDS property :
$|A| + |MC(A)| \in \{0, 5, 6, 7, 8\}$
(for diffusion of active cells)

Definition of the SR-MC constraint

$\forall j \in [0; 3]$:
$\sum_{k=0}^{3} \Delta X_i[(k + j)\%4][k] + \Delta Y_i[j][k] \in \{0, 5, 6, 7, 8\}$

CP Model for Step 1

- Impose constraints for all operations having an effect on the the truncated differences
- Impose additional constraints (at least one active byte)
- Set the objective function to minimize the number of active S-boxes

Problem

Too many inconsistent solutions!

CP Model for Step 1

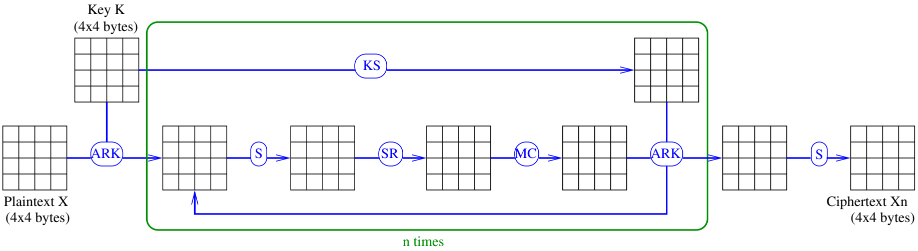
Reduce the number of inconsistent solutions

- Take the equality relationship into consideration : when $A == B$, $A \oplus B == 0$
- Consider the MDS property of two different columns

The Minizinc Code

`http://www.gerault.net/resources/CP_AES.tar.gz`

CP Model for Step 2



- Introduce a variable for every byte, whose domain is $\{0, 255\}$
- Impose the constraints of the differential distribution table, XOR etc. as table constraints
- Impose constraints according to the truncated differential characteristic

The Choco Code

`http://www.gerault.net/resources/Step2_AES.tar.gz`

Results for AES-128

- We find 19 truncated related-key differential characteristics with 20 active S-boxes in 7 hours, but none of them can be instantiated with an actual differential characteristic.
- We then find 1542 ones with 21 active S-boxes in around 12 hours. Among these, only 20 of them can be instantiated with actual differential characteristics.
- The probability of the optimal characteristic is 2^{-131}.

Round	$\delta X_i = X_i \oplus X_i'$	$\delta K_i = K_i \oplus K_i'$	Pr(States)	Pr(Key)
init.	366d1b80 dc37dbdb 9bc08d5b 00000000			
$i = 0$	00000000 71000000 00004d00 00000000	366d1b80 ad37dbdb 9bc0c05b 00000000	$2^{-6.2}$	–
1	b6f60000 009a0000 009a0000 009a0000	366d1b80 9b5ac05b 009a0000 009a0000	$2^{-7.2} \cdot 2^{-6.3}$	2^{-6}
2	00000000 009a0000 00000000 009a0000	ed6d1b80 7637dbdb 76addbdb 7637dbdb	$2^{-6.2}$	$2^{-6} \cdot 2^{-7.3}$
3	00000000 009a0000 009a0000 00000000	76addbdb 009a0000 7637dbdb 00000000	$2^{-6.2}$	–
4	00000000 009a0000 00000000 00000000	76addbdb 7637dbdb 00000000 00000000	2^{-6}	–
5	00000000 009a0000 009a0000 009a0000	76addbdb 009a0000 009a0000 009a0000	$2^{-6.3}$	2^{-6}
End/6	db000000 db9a0000 db000000 ad37dbdb	adaddbdb ad37dbdb adaddbdb ad37dbdb	–	–

TABLE – The optimal characteristic

TABLE – A comparison between the results obtained by CP and the graph-based search algorithm [Pierre-Alain Fouque, Jérémy Jean and Thomas Peyrin, CRYPTO 2013].

Rounds	Constraint Programming		Graph Search	
	#AS	Prob.	#AS	Prob.
3	5	2^{-31}	5	2^{-31}
4	12	2^{-79}	13	2^{-81}
5	17	2^{-105}	17	2^{-105}
6	21	2^{-131}	-	-

Search for Impossible differential and Zero-correlation Linear Approximation

Related work

- [Yu Sasaki and Yosuke Todo, EUROCRYPT 2017]
- [Cui, Jia, Fu, Chen and Wang, IACR ePrint 2016/689]

- Choose an input-output difference pattern (α, β).
- Construct a CP model $\mathcal{M}_{(\alpha,\beta)}$ whose solution set includes all valid differential characteristics.
- Solve $\mathcal{M}_{(\alpha,\beta)}$. If $\mathcal{M}_{(\alpha,\beta)}$ is infeasible, (α, β) is an impossible differential.
- Choose another (α, β) and repeat.

Search for Integral Distinguishers based on Bit-based Dvision Property

- Division property was proposed by Todo [Todo, EUROCRYPT 2015] which was extended to Bit-based division property [Todo and Morii, FSE 2016].

Bit-based division property

Let $\mathbb{X}$ be a multiset whose elements belong to $\mathbb{F}_2^n$. When the multiset $\mathbb{X}$ has the division property $\mathcal{D}_{\mathbb{K}}^{1^n}$, where $\mathbb{K}$ denotes a set of n-dimensional vectors in $\{0,1\}^n \subseteq \mathbb{Z}^n$, it fulfills the following condition

$$\bigoplus_{\mathbf{x}\in\mathbb{X}} x_0^{u_0} x_1^{u_1} \cdots x_{n-1}^{u_{n-1}} = \begin{cases} \text{unknown} & \text{if there are } \mathbf{k} \in \mathbb{K}, \text{s.t.} \mathbf{u} \succcurlyeq \mathbf{k} \\ 0 & \text{otherwise} \end{cases}$$

where $\mathbf{u} = (u_0, u_1, \cdots, u_{n-1}) \in \{0,1\}^n \subseteq \mathbb{Z}^n$, $\mathbf{x} = (x_0, x_1, \cdots, x_{n-1}) \in \mathbb{F}_2^n$.

Using Division Property

- Construct an input set with division property $\mathcal{D}^{1^n}_{\mathbb{K}}$.
- Propagate it against the target cipher to get the output set with division property $\mathcal{D}^{1^n}_{\mathbb{K}'}$
- Extract some useful integral property from $\mathcal{D}^{1^n}_{\mathbb{K}'}$

The rule of propagation

The propagation of the division property can be described as a set of bit vectors, which in turn can be modeled by the language of CP.

Propagation of Division Property against Vectorial Boolean Functions

Algorithm 1: propagate() Compute the output division property.

Input: A vectorial boolean function $\mathbf{f} : \mathbb{F}_2^m \rightarrow \mathbb{F}_2^n$, and an input pattern $\mathbf{u} = (u_0, \cdots, u_{m-1}) \in \mathbb{F}_2^m$, where $f(\mathbf{x}) = (f_0(\mathbf{x}), \cdots, f_{n-1}(\mathbf{x}))$ and $\mathbf{x} = (x_0, \cdots, x_{m-1})$;

Output: $\mathcal{O}$: a set of patterns $\mathbf{v} \in \mathbb{F}_2^n$ describing the division property of the output set;

$\mathcal{O} = \emptyset$;
if $\mathbf{u} = (0, \cdots, 0)$ **then**
    return $\mathcal{O} = \{(0, \cdots, 0)\}$
else
    for $\mathbf{v} \in \mathbb{F}_2^n/(0, \cdots, 0)$ **do**
        Let $F = \prod_{j=0}^{n-1} f_j^{v_j}(x_0, \cdots, x_{n-1})$;
        if $\prod_{j=0}^{m-1} x_j^{u_j} \lessdot F$ **then**
            $\mathcal{O} = \mathcal{O} \cup \{v\}$;
        end
    end
end
return reduced($\mathcal{O}$);

- [Xiang, Zhang, Bao and Lin, ASIACRYPT 2016]
- [Christina Boura and Anne Canteaut, CRYPTO 2016]
- [Ling Sun and Meiqin Wang, IACR ePrint 2016/392]

Example : the PRESENT S-box

Table: Division Trails of PRESENT Sbox

Input $\mathcal{D}_k^{1,4}$	Output $\mathcal{D}_{\mathbb{K}}^{1,4}$
(0,0,0,0)	(0,0,0,0)
(0,0,0,1)	(0,0,0,1) (0,0,1,0) (0,1,0,0) (1,0,0,0)
(0,0,1,0)	(0,0,0,1) (0,0,1,0) (0,1,0,0) (1,0,0,0)
(0,0,1,1)	(0,0,1,0) (0,1,0,0) (1,0,0,0)
(0,1,0,0)	(0,0,0,1) (0,0,1,0) (0,1,0,0) (1,0,0,0)
(0,1,0,1)	(0,0,1,0) (0,1,0,0) (1,0,0,0)
(0,1,1,0)	(0,0,0,1) (0,0,1,0) (1,0,0,0)
(0,1,1,1)	(0,0,1,0) (1,0,0,0)
(1,0,0,0)	(0,0,0,1) (0,0,1,0) (0,1,0,0) (1,0,0,0)
(1,0,0,1)	(0,0,1,0) (0,1,0,0) (1,0,0,0)
(1,0,1,0)	(0,0,1,0) (0,1,0,0) (1,0,0,0)
(1,0,1,1)	(0,0,1,0) (0,1,0,0) (1,0,0,0)
(1,1,0,0)	(0,0,1,0) (0,1,0,0) (1,0,0,0)
(1,1,0,1)	(0,0,1,0) (0,1,0,0) (1,0,0,0)
(1,1,1,0)	(0,1,0,1) (1,0,1,1) (1,1,1,0)
(1,1,1,1)	(1,1,1,1)

```
Tuples integral_path = new Tuples(true);
integral_path.add(0, 0, 0, 0, 0, 0, 0, 0);
integral_path.add(0, 0, 0, 1, 0, 0, 0, 1);
integral_path.add(0, 0, 0, 1, 0, 0, 1, 0);
integral_path.add(0, 0, 0, 1, 0, 1, 0, 0);
integral_path.add(0, 0, 0, 1, 1, 0, 0, 0);
integral_path.add(0, 0, 1, 0, 0, 0, 0, 1);
integral_path.add(0, 0, 1, 0, 0, 0, 1, 0);
integral_path.add(0, 0, 1, 0, 0, 1, 0, 0);
integral_path.add(0, 0, 1, 0, 1, 0, 0, 0);
integral_path.add(0, 0, 1, 1, 0, 0, 1, 0);
integral_path.add(0, 0, 1, 1, 0, 1, 0, 0);
integral_path.add(0, 0, 1, 1, 1, 0, 0, 0);
integral_path.add(0, 1, 0, 0, 0, 0, 0, 1);
integral_path.add(0, 1, 0, 0, 0, 0, 1, 0);
integral_path.add(0, 1, 0, 0, 0, 1, 0, 0);
integral_path.add(0, 1, 0, 0, 1, 0, 0, 0);
integral_path.add(0, 1, 0, 1, 0, 0, 1, 0);
integral_path.add(0, 1, 0, 1, 0, 1, 0, 0);
integral_path.add(0, 1, 0, 1, 1, 0, 0, 0);
integral_path.add(0, 1, 1, 0, 0, 0, 0, 1);
integral_path.add(0, 1, 1, 0, 0, 0, 1, 0);
integral_path.add(0, 1, 1, 0, 1, 0, 0, 0);
integral_path.add(0, 1, 1, 1, 0, 0, 1, 0);
integral_path.add(0, 1, 1, 1, 1, 0, 0, 0);
integral_path.add(1, 0, 0, 0, 0, 0, 0, 1);
integral_path.add(1, 0, 0, 0, 0, 0, 1, 0);
integral_path.add(1, 0, 0, 0, 0, 1, 0, 0);
integral_path.add(1, 0, 0, 0, 1, 0, 0, 0);
integral_path.add(1, 0, 0, 1, 0, 0, 1, 0);
integral_path.add(1, 0, 0, 1, 0, 1, 0, 0);
integral_path.add(1, 0, 0, 1, 1, 0, 0, 0);
integral_path.add(1, 0, 1, 0, 0, 0, 1, 0);
integral_path.add(1, 0, 1, 0, 0, 1, 0, 0);
integral_path.add(1, 0, 1, 0, 1, 0, 0, 0);
integral_path.add(1, 0, 1, 1, 0, 0, 1, 0);
integral_path.add(1, 0, 1, 1, 0, 1, 0, 0);
integral_path.add(1, 0, 1, 1, 1, 0, 0, 0);
```

Propagation of Division Property : Division Trail

- The bit-based division property can be described by the propagation of bit patterns with some special meaning, which leads to the concept of *division trail*.

Division Trail [Xiang, Zhang, Bao and Lin, ASIACRYPT 2016]

Let $\mathcal{F}$ be the round function of an iterated block cipher. Assume that the input multi-set to the block cipher has initial division property $\mathcal{D}^{1^n}_{\mathbb{K}_0}$ with $\mathbb{K}_0 = \{\mathbf{k}\}$. This initial division property propagates through the round function which forms a chain

$$\mathcal{D}^{1^n}_{\mathbb{K}_0} \xrightarrow{\mathcal{F}} \mathcal{D}^{1^n}_{\mathbb{K}_1} \xrightarrow{\mathcal{F}} \mathcal{D}^{1^n}_{\mathbb{K}_2} \xrightarrow{\mathcal{F}} \cdots$$

For any vector $\mathbf{k}^*_i \in \mathbb{K}_i (i \geq 1)$, there must exist a vector $\mathbf{k}^*_{i-1}$ in $\mathbb{K}_{i-1}$ such that $\mathbf{k}^*_{i-1}$ can propagate to $\mathbf{k}^*_i$ according to the rules of division property propagation. Furthermore, for $(\mathbf{k}_0, \mathbf{k}_1, \cdots, \mathbf{k}_r) \in \mathbb{K}_0 \times \mathbb{K}_1 \times \cdots \times \mathbb{K}_r$, if $\mathbf{k}_{i-1}$ can propagate to $\mathbf{k}_i$ for all $i \in \{1, 2, \cdots, r\}$, we call $(\mathbf{k}_0, \mathbf{k}_1, \cdots, \mathbf{k}_r)$ an r-round division trail.

The rule for detecting integral distinguisher based on division property

Set without Integral Property

Let $\mathbb{X}$ be a multiset with division property $\mathcal{D}_{\mathbb{K}}^{1^n}$, then $\mathbb{X}$ does not have integral property if and only if $\mathbb{K}$ contains all the n unit vectors.

- Construct a CP model $\mathcal{M}_{\mathbf{e}_j}$ whose solution set contains all the division trails whose output division property is set to $\mathbf{e}_j$.
- If we can find at least one $\mathcal{M}_{\mathbf{e}_j}$ for $j \in \{0, \cdots, n-1\}$ which is infeasible, then we find an integral distinguisher.

Accelerating the Search

- Ordering heuristic
  - The order in which the variables are assigned has significant impact on the efficiency of the resolution.
  - We choose the generic ordering heuristic called domain over weighted degree [Frédéric Boussemart et al., ECAI 2004]
- Random restart

Results on PRESENT, HIGHT, and SKINNY

- Retrieve the 9-round distinguisher of PRESENT found by MILP method(cost 3.4 minutes) in 36 seconds.
- Rediscover all zero-correlation linear approximations of the 17-round in 1709 seconds (MILP cost 4786).
- SKINNY : We found 16 impossible differentials leading to 18-round attack. Better results obtained by other researchers are now available for SKINNY [IACR ePrint 2016/1127, 1120, 1115, and 1108]

Note

During the process of designing new ciphers, the evaluation sometimes needs to be repeated several times. Hence, even though not crucial, a good CPU time is a desirable feature.

Comparing Solvers

- Pick two problems as benchmark
  - Optimization : find the best trail of PRESENT
  - Enumeration : list all solutions in a given linear hull of PRESENT
- Solvers
  - MILP solvers : Gurobi, SCIP
  - CP solvers : Choco, Chuffed, PICAT_SAT

Comparing Solvers

TABLE – Optimization problem, with a time limit of 2 hours.

Rounds	Prob.	Time by Gurobi (sec.)	Time by Choco (sec.)	Time by Chuffed (sec.)	Time by PICAT_SAT (sec.)
3	2^{-8}	2	4.1	0.2	12.8
4	2^{-12}	25	750.8	11.4	22.5
5	2^{-20}	453	-	3404.5	91.4
6	2^{-24}	2184	-	-	486.2
7	2^{-28}	-	-	-	5883.9

Comparing Solvers

TABLE – Enumerating the linear hull of PRESENT

Rounds	Time by SCIP (sec.)	Number of solutions by SCIP	Time by Choco (sec.)	Number of solutions by Choco
4	0.1	3	0.023	3
5	0.28	17	0.031	17
6	37.7	8064	0.359	8064

Conclusion and Discussion

- CP is indeed a convenient tool for symmetric-key cryptanalysis
  - Easy to implement
  - Sometimes faster
- Further directions
  - Most automatic tools focus on the search for distinguishers
  - Can we automate the key-recovery part ?
    [Patrick Derbez and Pierre-Alain Fouque, CRYPTO 2016]
    [Li Lin, Wenling Wu, Yafei Zheng, FSE 2016]

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Thanks for your attention !