General Diffusion Analysis: How to Find Optimal Permutations for Generalized Type-II Feistel Schemes

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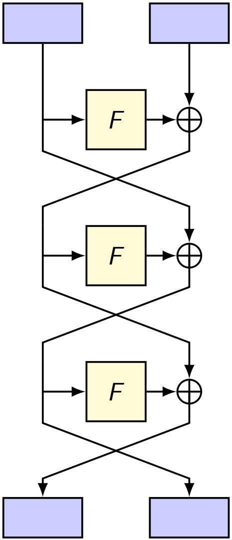
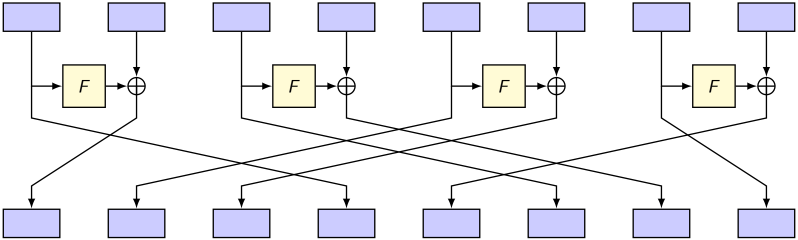
Fast Software Encryption — 2019-03-26 — Paris, France

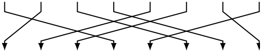
The Feistel Network



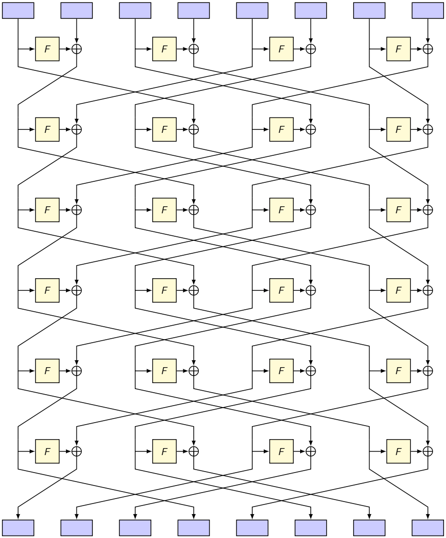
- How to construct a permutation?
- Challenging task
- Split the problem in half and iterate
- DES, Camellia, Simon, . . .

Type-II Generalized Feistel Structure (GFS)



- Split into k blocks
- $k/2$ parallel mini-Feistel functions F (easier to design)
- Then a block-wise permutation $\pi \in \mathcal{S}_k$: 
- CLEFIA ($k = 4$), Simpara ($k = 4, 6, 8$), TWINE ($k = 16$)
- Problem: Diffusion needs more rounds

Maximum Diffusion Round (DR)



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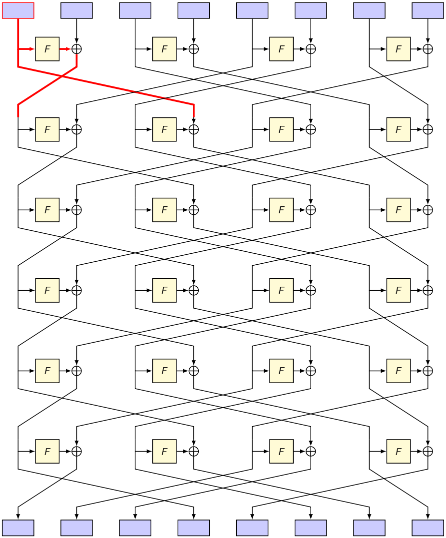

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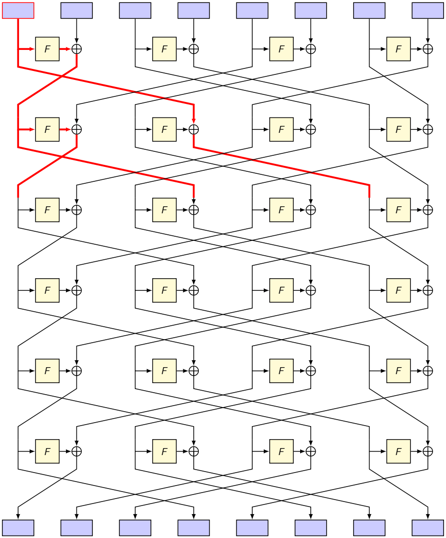

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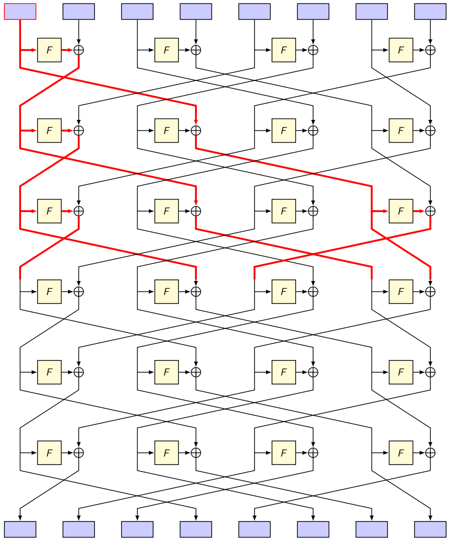

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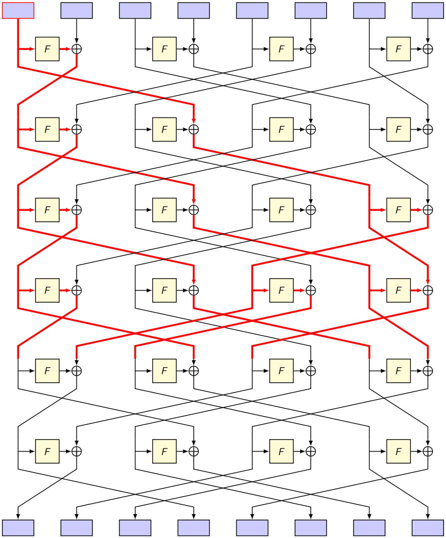

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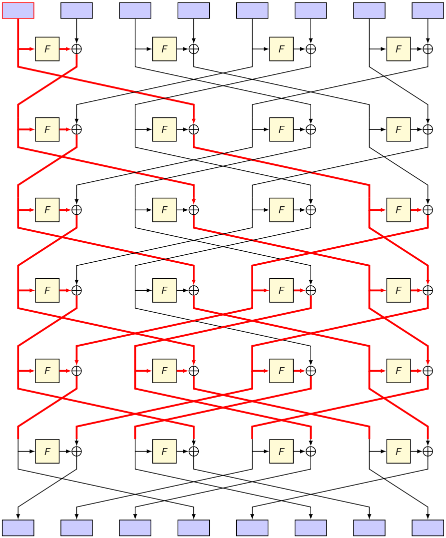

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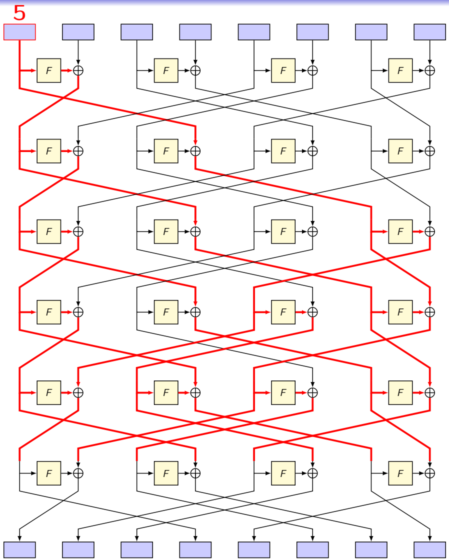

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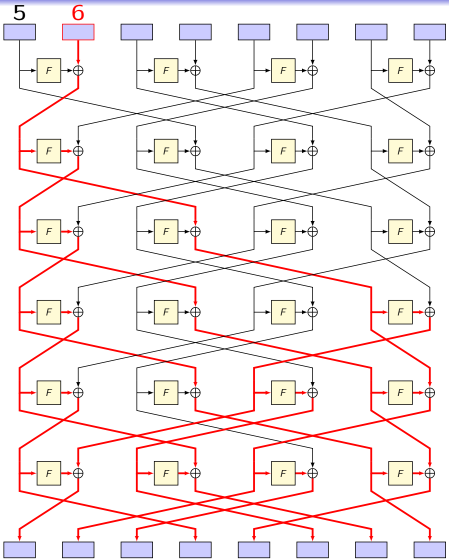

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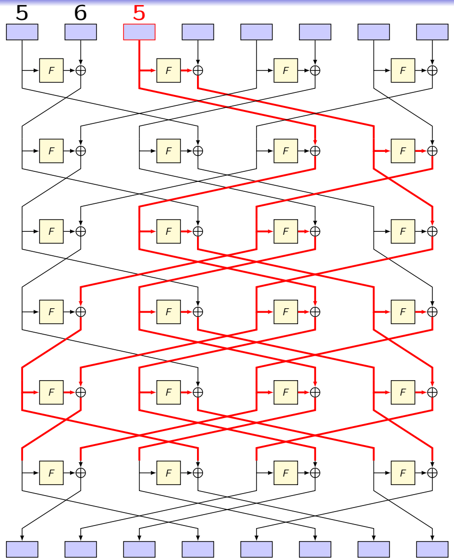

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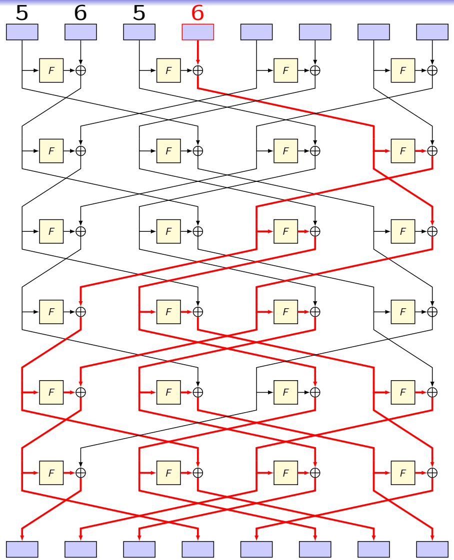

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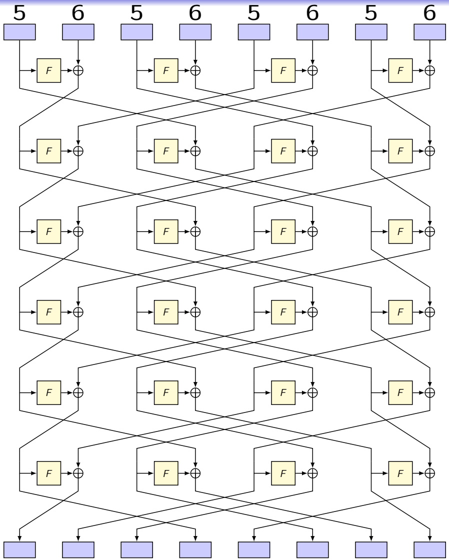

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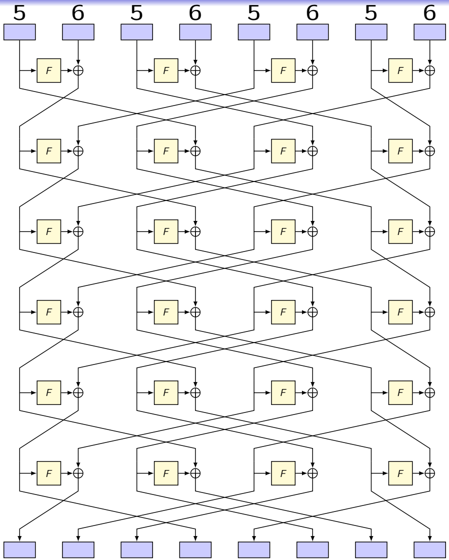
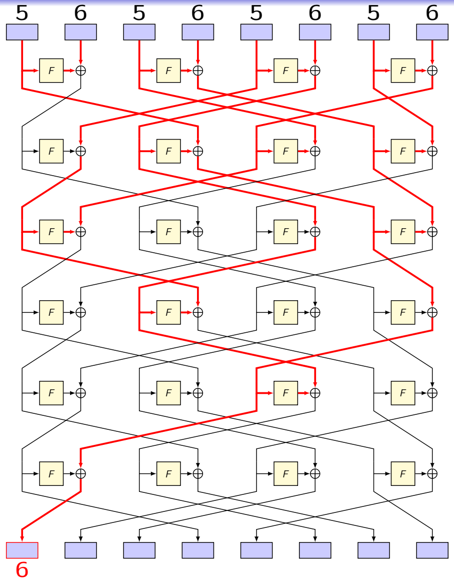


- Simple criterion
- Depends only on the permutation π
- Link with impossible differential and saturation attacks
- Encryption

Maximum Diffusion Round (DR)



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  AND
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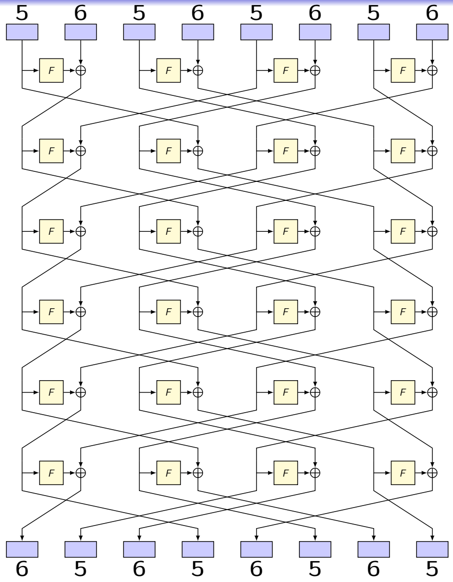

- Simple criterion
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- Encryption
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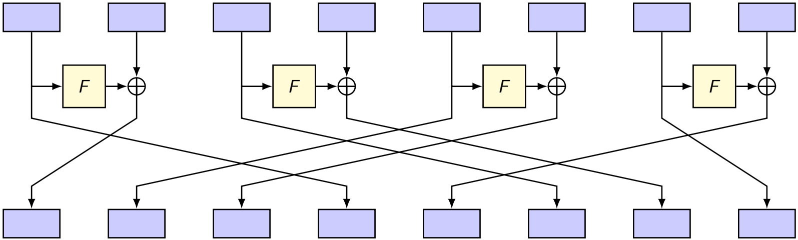
- here $DR(\pi) = 6$

Previous Works



- Suzaki and Minematsu, FSE 2010
- Focus on even-odd GFS ($\mathcal{S}_k^{eo}$)
- Exhaustive search for $k \leq 16$ blocks
- Power of two case : generic construction in $DR(\pi) = 2\log_2 k$

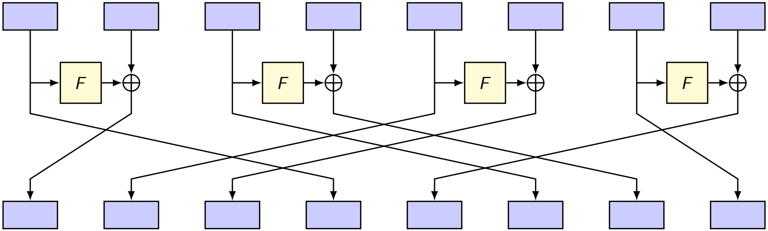
k	DR(rot)	min $DR(\pi)$
4	4	4
6	6	5
8	8	6
10	10	7
12	12	8
14	14	8
16	16	8

Our Contributions

- Constructive upper-bound on the number of even-odd GFS up to equivalence
- Exhaustive search for $k \leq 24$
- New criterion to reduce search space: Collision-free depth
- Power of two case: new permutations based on graph coloring
- Case of non even-odd permutations

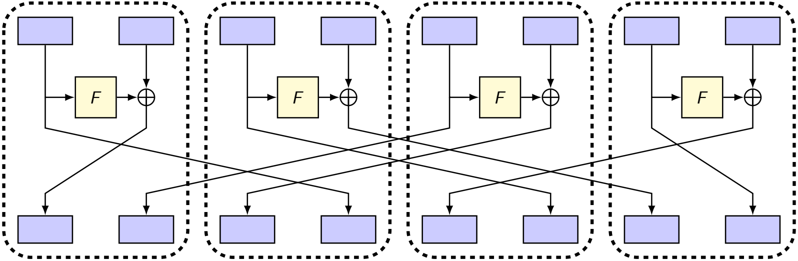
k	[SM10]	this paper
4	4	4
6	5	5
8	6	6
10	7	7
12	8	8
14	8	8
16	8	8
18		8
20		9
22		8
24		9
26		9
32	10	10
64	12	11
128	14	13

Equivalence of GFS



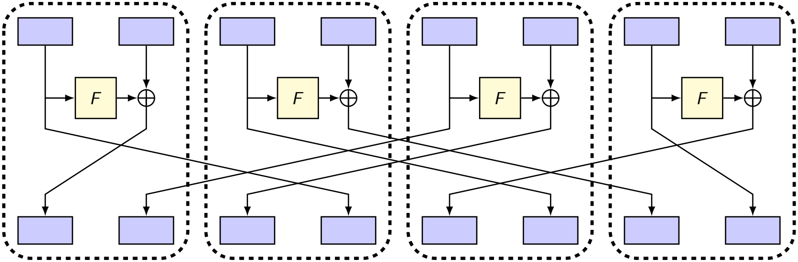
- Equivalence up to block reindexing

Equivalence of GFS



- Equivalence up to block reindexing
- Permutations of pairs: swap blocks in a pair-wise manner

Equivalence of GFS



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- Permutations of pairs: swap blocks in a pair-wise manner

$$\mathcal{S}_k^p := \{\varphi \in \mathcal{S}_k | \forall i \leq \frac{k}{2} - 1,\ \varphi(2i) \text{ is even and } \varphi(2i+1) = \varphi(2i) + 1\}$$

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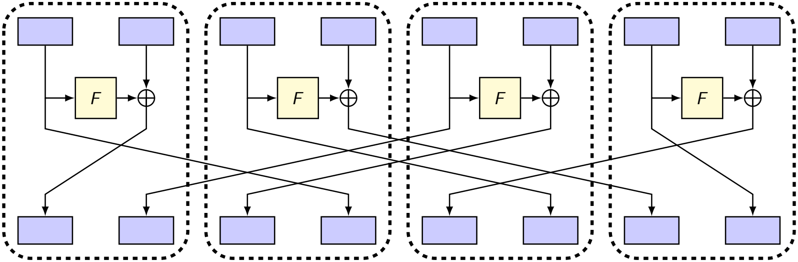

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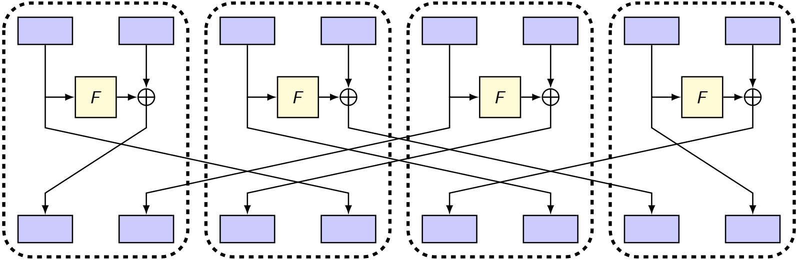
- "Pair-equivalence": $\mathcal{S}_k^p$ acts on $\mathcal{S}_k$ by conjugation

$$\pi_1 \equiv \pi_2 \text{ iff } \exists \varphi \in \mathcal{S}_k^p \text{ s.t. } \pi_1 = \varphi \circ \pi_2 \circ \varphi^{-1}$$

Number of Even-odd GFS up to Pair-equivalence



Bijection: $\begin{cases} \mathcal{S}_{k/2} \times \mathcal{S}_{k/2} \rightarrow \mathcal{S}_k^{eo} \text{ (even-odd GFS)} \end{cases}$

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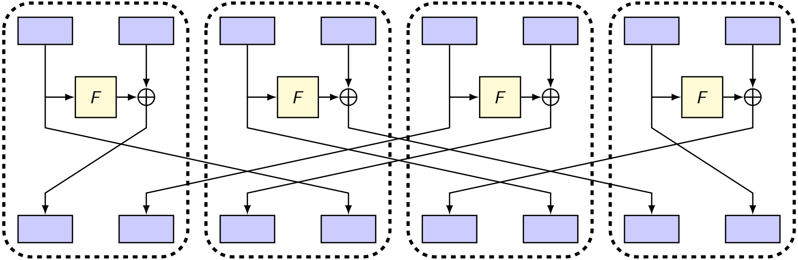

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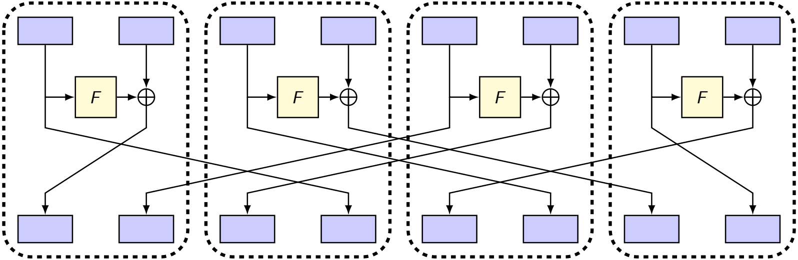

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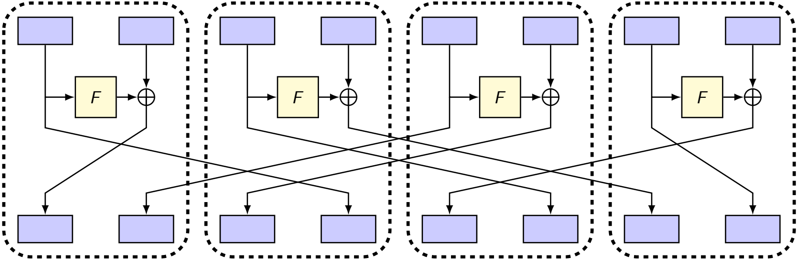

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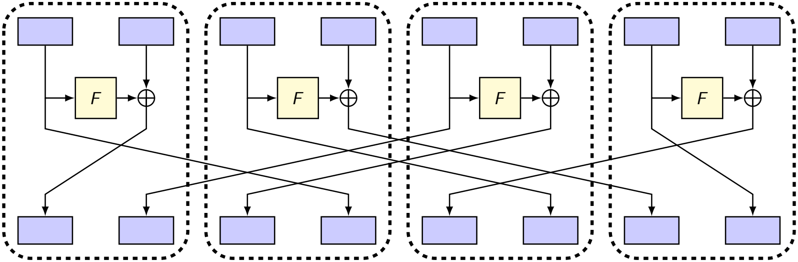

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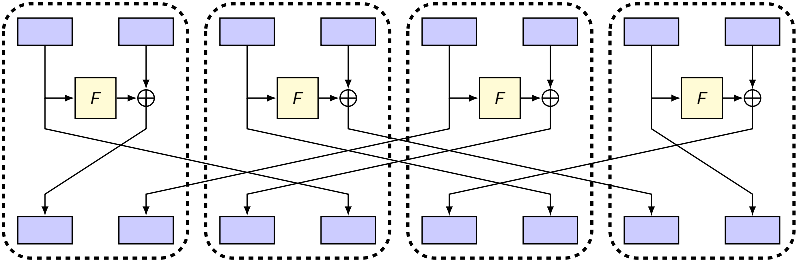

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- Number of conjugacy class in $\mathcal{S}_{k/2}$: $N_{k/2} = \mathcal{O}(e^{\pi\sqrt{k/3}})$

Exhaustive search of Optimal Even-odd GFS

k		min $DR(\pi)$	Number of classes
6		5	1
8		6	2
10		7	3
12		8	32
14		8	23
16		8	13
18		8	2
20		9	2133
22		8	4
24		9	56

Exhaustive search of Optimal Even-odd GFS

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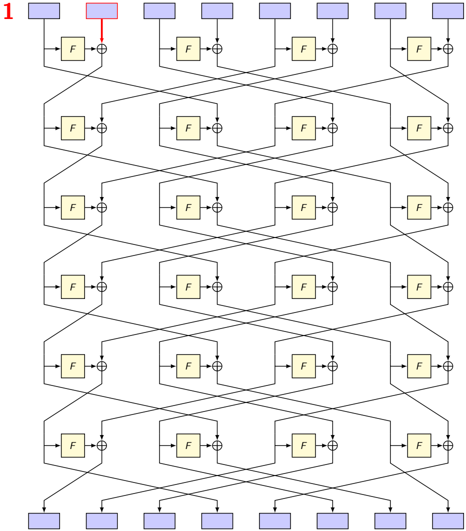
- What happens with $k = 18$, 20 and 22?

Exhaustive search of Optimal Even-odd GFS

k	lower bound	min $DR(\pi)$	Number of classes
6	5	5	1
8	6	6	2
10	6	7	3
12	7	8	32
14	7	8	23
16	7	8	13
18	8	8	2
20	8	9	2133
22	8	8	4
24	8	9	56

- What happens with $k = 18$, 20 and 22? $\longrightarrow$ lower bound

Lower Bound on the Diffusion Round of Type-II GFS



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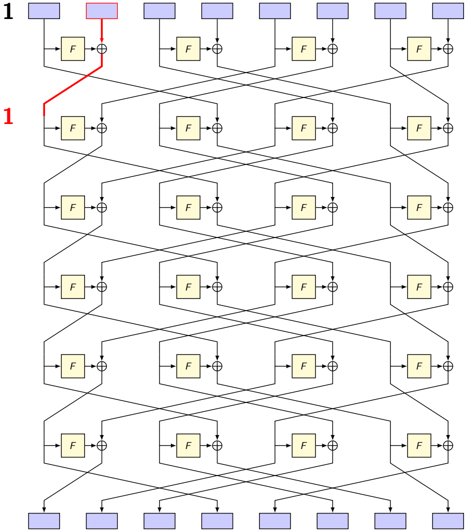

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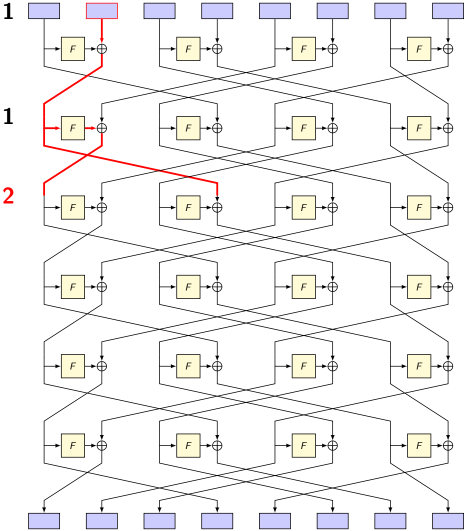

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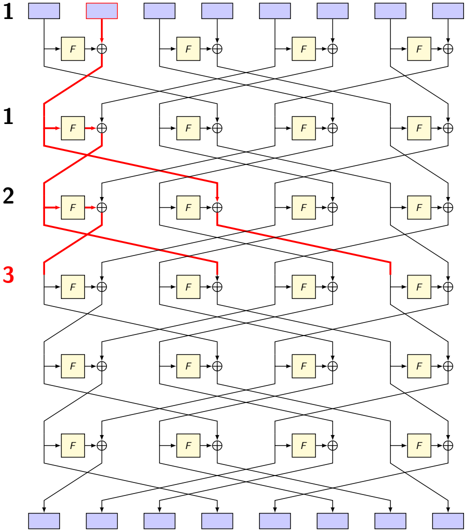

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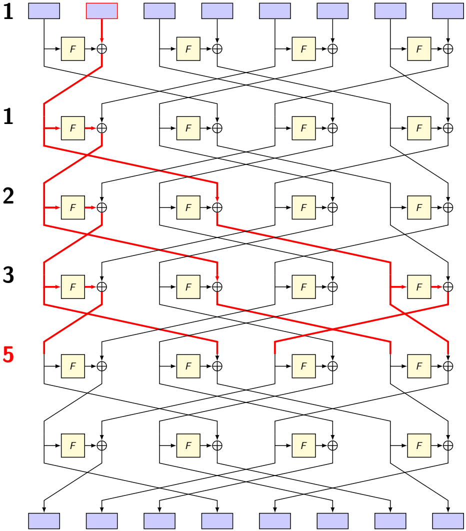

- Fibonacci Sequence

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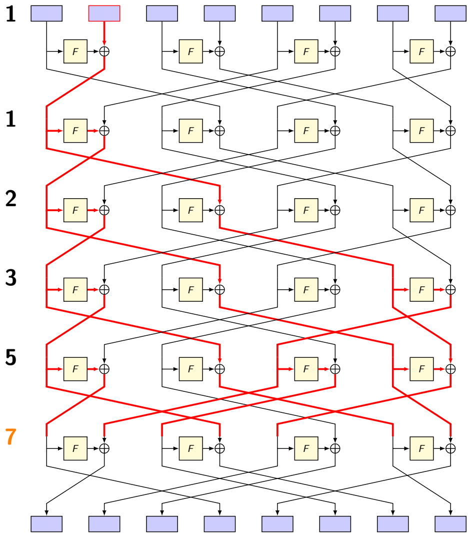

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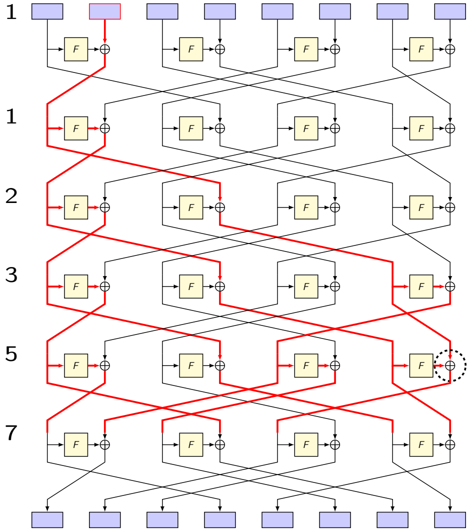

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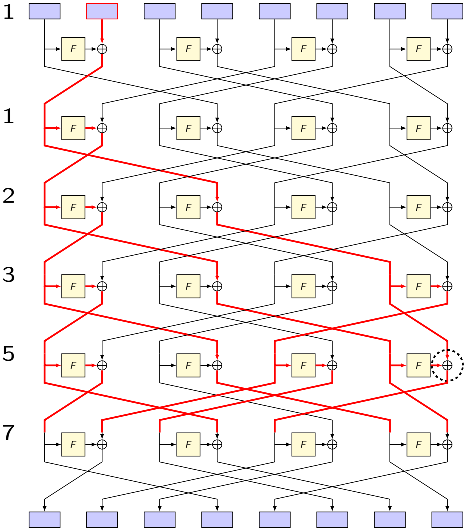
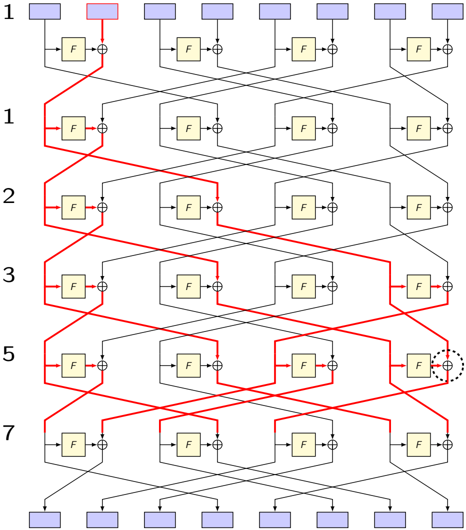


- Fibonacci Sequence
- Until Collision
- Lower Bound: no collision happens

$$2 \cdot Fib(DR(\pi)) \geq k$$

New Idea for finding good GFS



- $k > 26 \longrightarrow$ exhaustive search intractable

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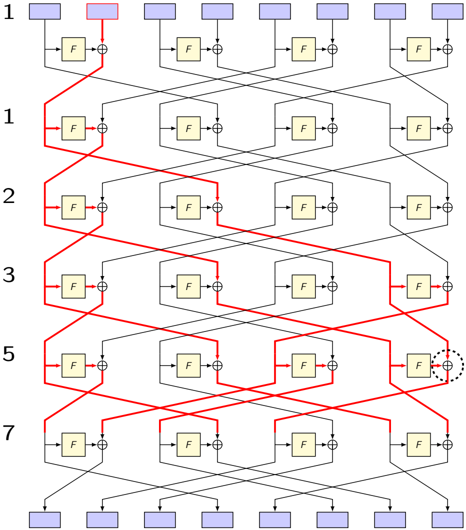

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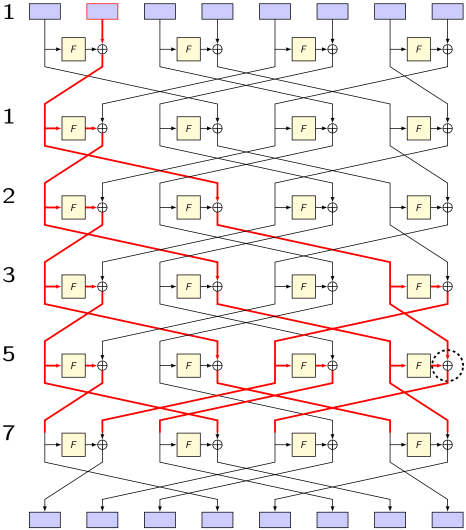

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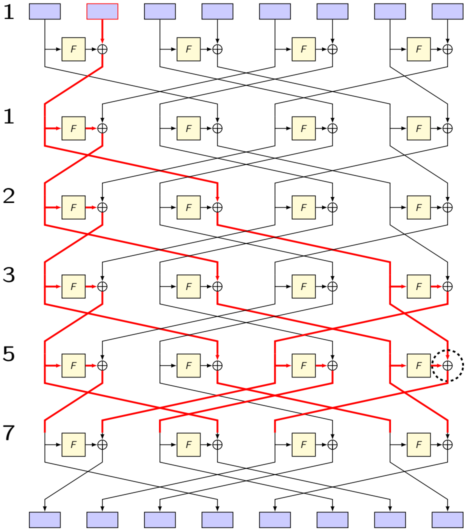

- $k > 26 \longrightarrow$ exhaustive search intractable
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- New criterion: $CD(\pi)$ Collision-free depth
- here $CD(\pi) = 3$

Collision-free Depths of Optimal Even-odd Permutations

k	16		18	20				22	24		
min $DR(\pi)$	8		8	9				8	8		
bound on CD	4		5	5				5	5		
$CD(\pi)$	3	4	3	2	3	4	5	5	3	4	5
$\sharp$ classes	9	4	2	165	1624	340	4	4	19	32	5

- Tradeoff between search space size and number of results

Interresting Case: $k = 26$

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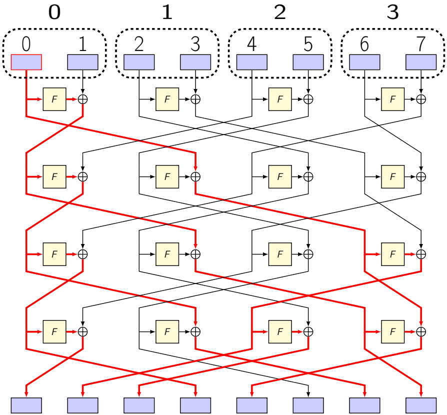
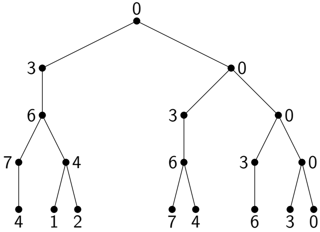
$\Rightarrow$ This MUST be optimal

Tree and Block-Tree Representation





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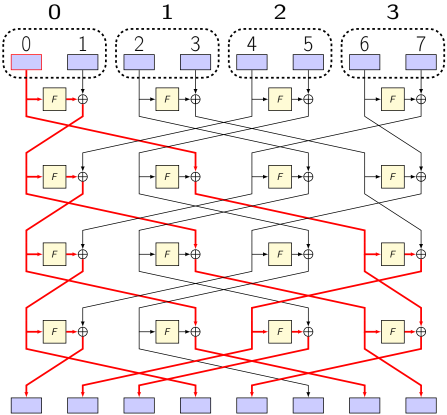
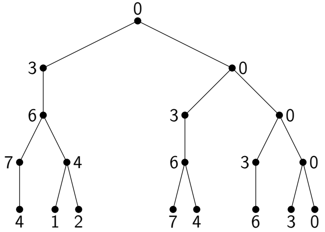
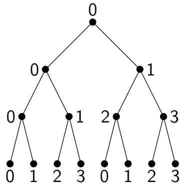
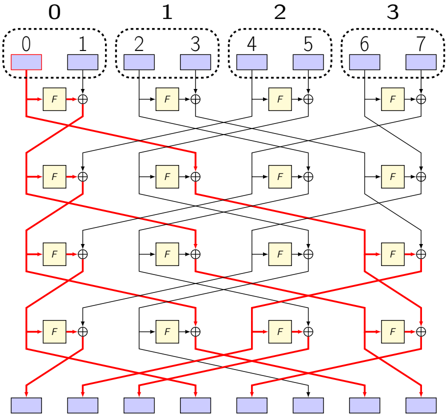
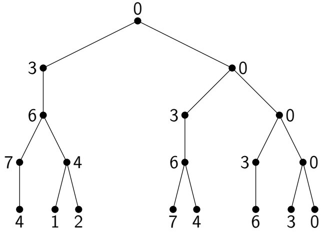


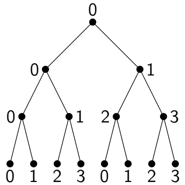




Tree and Block-Tree Representation

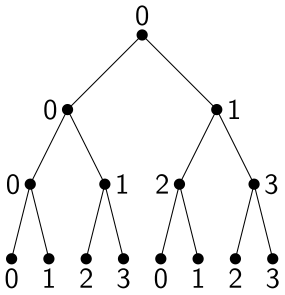






- Grouping blocks by pair is used in [SM10]

Pros and Cons of Block-Tree Representation



Pros:

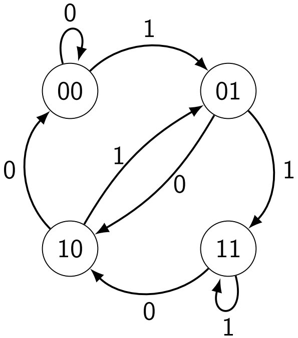
- Balanced Binary Tree
- Fewer nodes
- Simpler structure

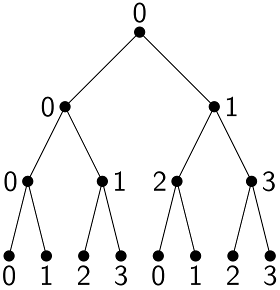
Cons:

- Does not uniquely represent a permutation π
- Where do even and odd nodes go within a block?

$\longrightarrow$ Need to specify it as an edge-colouring

Candidates yielding good GFS?





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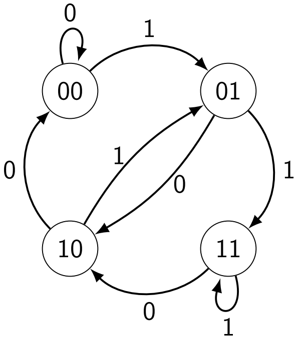

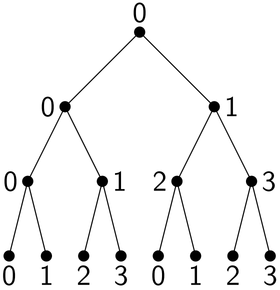

- De Bruijn graph used by [SM10]
- Fill with the least possible value

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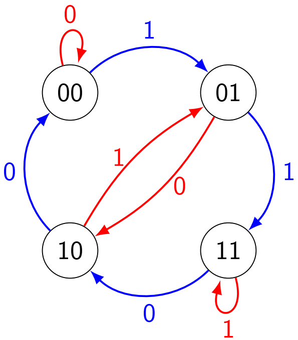

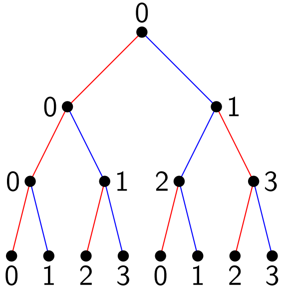

- De Bruijn graph used by [SM10]
- Fill with the least possible value
- Many ways to color the graph
- Exhaust in $\mathcal{O}(2^{k/4})$

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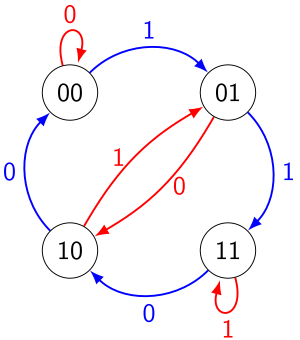

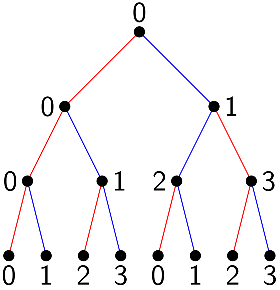

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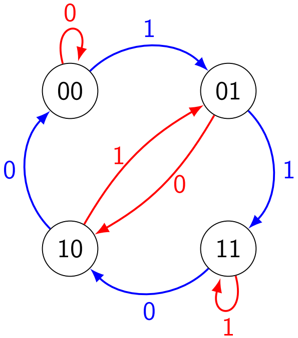

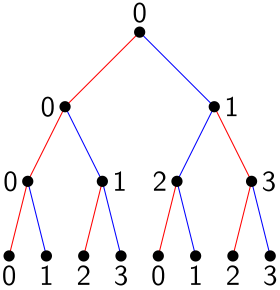

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k	[SM10]	this paper
8	6	6
16	8	8
32	10	10
64	12	11
128	14	13

Until now...

- Constructive upper-bound on the number of even-odd GFS up to equivalence
- Exhaustive search for $k \leq 24$
- New criterion to reduce search space: Collision-free depth
- Power of two case: new permutations based on graph coloring
- Case of non even-odd permutations

k	[SM10]	this paper
4	4	4
6	5	5
8	6	6
10	7	7
12	8	8
14	8	8
16	8	8
18		8
20		9
22		8
24		9
26		9
32	10	10
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128	14	13

Number of non even-odd Permutations Classes

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$$(k/2)! \leq \text{ number of classes } \leq N_{k/2} \cdot (k/2)!$$

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- $\frac{k!}{(k/2)!}$: Right-coset representatives of $\mathcal{S}_k \mod \mathcal{S}_k^p$
- Exhaustive search $k \leq 20$: no result better than even-odds

Conclusion

- Study of type-II Generalized Feistel Structures
- Permutations up to pair-equivalence
- Constructive upper-bound
- Exhaustive search up to $k \leq 24$ (even-odd) or $k \leq 20$ (general)
- Permutations with no collision in the early rounds
- Improved Results for $k = 64$ and 128

Thank you for your attention.

Do you have any questions?